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Formulation of the solar surface dynamo

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ABSTRACT

The solar surface dynamo has become an active area of research in an attempt to understand the origin of a variety of magnetic field structures on the sun. The major modification that needs to be incorporated in the standard dynamo process is the inclusion of the partial ionization of the gas in the layers underlying and overlaying the photosphere along with the effects associated with the multifluid nature of the system. This not only changes the inertia carrying species but also substantially modifies the temporal and spatial evolution of the magnetic induction. The energy equation also carries the import of these non-ideal effects. The effects such as the Hall effect and the ambipolar diffusion take the dynamo study beyond the realm of the ideal magnetohydrodynamics. In this paper, a first principle formulation of the solar surface dynamo problem has been attempted.

Key words: partially ionized plasma, surface dynamo, Hall effect, ambipolar diffusion .

1 INTRODUCTION

Vögler and Schüssler (2007) with their paper on solar surface dynamo have opened up a new area of the investigation of the solar magnetic field structures. The solar surface fields on relatively short spatial and time scales with varying geometries are now observed and inferred routinely. This necessitates the inclusion of the realistic solar atmospheric models in the dynamo mechanism. Vögler and Schüssler (2007) have studied the surface dynamo including the effects of stratification, compressibility, partial ionization, radiative transfer as well as

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an open lower boundary. While this is a major effort, the magnetic induction equation they have used is still bereft of the effects ensuing from the multifluid interactions in a partially ionized plasma specifically the Hall and the ambipolar diffusion effects. Recently Krishan and Gangadhara (2008) initiated a study of the meanfield dynamo in a partially ionized plasma including the Hall and the ambipolar effects in the induction equation for closed boundary conditions while neglecting stratification, compressibility and radiative transfer effects. In this paper an attempt is made to formulate the solar surface dynamo including the Hall effect and the ambipolar diffusion while retaining all the effects included in Vögler and Schüssler (2007). A single fluid description of the partially ionized plasma in the limit of weak ionization is presented in section 2. The mean-field dynamo equation in the turbulent partially ionized plasma is derived in section 3 and the expressions for the α and the β parameters of a meanfield dynamo are extracted. A crude estimate of the time scale of evolution of the magnetic field due to turbulent ambipolar diffusion arising due to the fluctuating ion-neutral collision frequency is given.

2 THREE-COMPONENT MAGNETOFLUID

We begin with the three component partially ionized plasma consisting of electrons (e), ions (i) of mass density ρ_i and neutral particles (n) of mass density ρ_n. A weakly ionized plasma is defined by the condition (Alfvén and Fälthammer 1962) that the electron-neutral collision frequency $\nu_{\rm en} \sim 10^{-15} n_{\rm n}\sqrt{8K_{\rm B}T/(\pi m_{\rm en})}$ is much larger than the electron-ion collision frequency $\nu_{\rm ei} \sim 6\times 10^{-24} n_{\rm i}\Lambda Z^2 (K_{\rm B}T)^{-3/2}$. This translates into the ionization fraction $n_{\rm e}/n_{\rm n} < 5\times 10^{-11} T^2$ (Alfvén and Fälthammer 1962) where n's are the particle densities and T is the temperature in Kelvin. A major part of the solar photosphere (Leake & Arber 2006; Krishan & Varghese 2007) qualifies to be a weakly ionized plasma. The equation of motion of the electrons can be written as:

$$m_{\rm e} n_{\rm e}\left[\frac{\partial \boldsymbol{V}_{\rm e}}{\partial t} + (\boldsymbol{V}_{\rm e}\cdot\nabla)\boldsymbol{V}_{\rm e}\right] = -\nabla p_{\rm e} - \\ e n_{\rm e}\left[\boldsymbol{E} + \frac{\boldsymbol{V}_{\rm e}\times\boldsymbol{B}}{c}\right] - m_{\rm e} n_{\rm e}\nu_{\rm en}(\boldsymbol{V}_{\rm e} - \boldsymbol{V}_{\rm n})\ . \qquad (1)$$

where the electron-ion collisions have been neglected since the ionized component is of low density. On neglecting the electron inertial force, the electric field $\boldsymbol{E}$ is found to be:

$$\boldsymbol{E} = -\frac{\boldsymbol{V}_{\rm e}\times\boldsymbol{B}}{c} - \frac{\nabla p_{\rm e}}{e n_{\rm e}} - \frac{m_{\rm e}}{e}\nu_{\rm en}(\boldsymbol{V}_{\rm e} - \boldsymbol{V}_{\rm n})\ . \qquad (2)$$

This gives us Ohm's law. For $\delta = (\rho_{\rm i}/\rho_{\rm n}) \ll 1$ along with the dominance of the ion-neutral collisional force over the inertial force, the ion dynamics can be ignored. The ion force balance then becomes:

$$0 = -\nabla p_{\rm i} + en_{\rm i}\left[\boldsymbol{E} + \frac{\boldsymbol{V}_{\rm i}\times\boldsymbol{B}}{c}\right] - \nu_{\rm in}\rho_{\rm i}(\boldsymbol{V}_{\rm i} - \boldsymbol{V}_{\rm n}) \ , \tag{3}$$

where $\nu_{\rm in}$ is the ion-neutral collision frequency, and the ion-electron collisions have been neglected for the low density ionized component. Substituting for $\boldsymbol{E}$ from Eq. (2) we find the relative velocity between the ions and the neutrals:

$$\boldsymbol{V}_{\rm n} - \boldsymbol{V}_{\rm i} = \frac{\nabla(p_{\rm i}+p_{\rm e})}{\nu_{\rm in}\rho_{\rm i}} - \frac{\boldsymbol{J}\times\boldsymbol{B}}{c\nu_{\rm in}\rho_{\rm i}} - \frac{m_{\rm e}\nu_{\rm en}}{e}\boldsymbol{J}, \tag{4}$$

where

$$\boldsymbol{J} = en_{\rm e}(\boldsymbol{V}_{\rm i} - \boldsymbol{V}_{\rm e}) \ . \tag{5}$$

and a term proportional to $(m_{\rm e}nu_{\rm en})/(m_{\rm i}\nu_{\rm in})$ has been neglected. The equation of motion of the neutral fluid is:

$$\begin{aligned}\rho_{\rm n}\left[\frac{\partial\boldsymbol{V}_{\rm n}}{\partial t} + (\boldsymbol{V}_{\rm n}\cdot\nabla)\boldsymbol{V}_{\rm n}\right] = {} & -\nabla p_{\rm n} - \nu_{\rm ni}\rho_{\rm n}(\boldsymbol{V}_{\rm n} - \boldsymbol{V}_{\rm i}) - \\ & \nu_{\rm ne}\rho_{\rm n}(\boldsymbol{V}_{\rm n} - \boldsymbol{V}_{\rm e}) + \rho_{\rm n}\boldsymbol{g} + \\ & \nabla.\boldsymbol{\tau} \ ,\end{aligned} \tag{6}$$

where $\boldsymbol{g}$ is the gravitational acceleration, $\boldsymbol{\tau}$ is the viscous stress tensor of the neutral fluid defined as

$$\tau_{\rm ij} = \mu\left(\frac{\partial V_{\rm ni}}{\partial x_{\rm j}} + \frac{\partial V_{\rm nj}}{\partial x_{\rm i}} - \frac{2}{3}\delta_{\rm ij}\left(\nabla.\boldsymbol{V}_{\rm n}\right)\right), \qquad i,j = 1,2,3. \ . \tag{7}$$

Substituting for $\boldsymbol{V}_{\rm n} - \boldsymbol{V}_{\rm i}$ from Eq. (4), and using $\nu_{\rm in}\rho_{\rm i} = \nu_{\rm ni}\rho_{\rm n}$ we find:

$$\begin{aligned}\rho_{\rm n}\left[\frac{\partial\boldsymbol{V}_{\rm n}}{\partial t} + (\boldsymbol{V}_{\rm n}\cdot\nabla)\boldsymbol{V}_{\rm n}\right] = -\nabla p + \frac{\boldsymbol{J}\times\boldsymbol{B}}{c} + \rho_{\rm n}\boldsymbol{g} + \\ \nabla.\boldsymbol{\tau}, \ .\end{aligned} \tag{8}$$

where $p = p_{\rm n} + p_{\rm i} + p_{\rm e}$ and μ is the dynamic viscosity. Observe that the neutral fluid is subjected to the Lorentz force as a result of the strong ion-neutral coupling due to their collisions. A comparison with the corresponding equation of Vögler and Schüssler (2007) shows that here the neutral mass density ρ_n appears instead of the total mass density. This is valid for $\rho_n >> \rho_i$ in a weakly ionized plasma.

Consider Faraday's law of induction:

$$\frac{\partial\boldsymbol{B}}{\partial t} = -c\nabla\times\boldsymbol{E} \tag{9}$$

By substituting for the electric field from Eq. (2), we get

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \left[\boldsymbol{V}_{\rm e} \times \boldsymbol{B} + \frac{c}{en_{\rm e}} \nabla p_{\rm e} + \frac{m_{\rm e} c \nu_{\rm en}}{e} \left(\boldsymbol{V}_{\rm e} - \boldsymbol{V}_{\rm n} \right) \right] . \tag{10}$$

Here $\eta = m_{\rm e}\nu_{\rm en}c^2/(4\pi e^2 n_{\rm e})$ is the electrical resistivity predominantly due to electron-neutral collisions. Using the construction

$$\boldsymbol{V}_{\rm e} \times \boldsymbol{B} = [\boldsymbol{V}_{\rm n} - (\boldsymbol{V}_{\rm n} - \boldsymbol{V}_{\rm i}) - (\boldsymbol{V}_{\rm i} - \boldsymbol{V}_{\rm e})] \times \boldsymbol{B} , \tag{11}$$

substituting for the relative velocity of the ion and the neutral fluids from Eq. (4), assuming $p_{\rm e} = p_{\rm i}$ and neglecting terms of the order $(m_{\rm e}\nu_{\rm en})/(m_{\rm i}\nu_{\rm in})$ Eq. (10) becomes:

$$\begin{aligned} \frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \left[\left(\boldsymbol{V}_{\rm n} - \frac{\boldsymbol{J}}{en_{\rm e}} + \frac{\boldsymbol{J} \times \boldsymbol{B}}{c\nu_{\rm in}\rho_{\rm i}} \right) \times \boldsymbol{B} \right] + \\ \frac{c}{en_{\rm e}} \left(\nabla p_{\rm i} - 2 \frac{\nabla p_{\rm i} \times \boldsymbol{\Omega}_{\rm ic}}{\nu_{\rm in}} \right) + \\ \eta \nabla^2 \boldsymbol{B} . \end{aligned} \tag{12}$$

where $\boldsymbol{\Omega}_{\rm ic} = \frac{e\boldsymbol{B}}{m_{\rm i}c}$ is the ion cyclotron frequency. One can easily identify the Hall term $(\boldsymbol{J}/en_{\rm e})$, and the ambipolar diffusion term $(\boldsymbol{J} \times \boldsymbol{B})$ (Brandenburg and Zweibel 1994, Chitre & Krishan 2001). The Hall term is much larger than the ambipolar term for large neutral particle densities or for $\nu_{\rm in} \gg \omega_{\rm ci}$. In this system the magnetic field is not frozen to any of the fluids. The induction equation carries the effects of stratification as well as the $\nabla p_{\rm i} \times \boldsymbol{B}$ fluid drift. In what follows the electron and the ion densities (pressures) are assumed to be equal and constant. We write the induction equation in a compact form as:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times [\boldsymbol{V}_E \times \boldsymbol{B} - \eta \nabla \times \boldsymbol{B}] , \tag{13}$$

where

$$\boldsymbol{V}_{\rm E} = \boldsymbol{V}_{\rm n} + \boldsymbol{V}_{\rm H} + \boldsymbol{V}_{\rm Am} \tag{14}$$

with

$$\boldsymbol{V}_{\rm H} = -\frac{\boldsymbol{J}}{en_{\rm e}} \tag{15}$$

as the Hall velocity and

$$\boldsymbol{V}_{\rm Am} = \frac{\boldsymbol{J} \times \boldsymbol{B}}{c\nu_{\rm in}\rho_{\rm i}} \tag{16}$$

could be called the ambipolar velocity. One can notice that the Hall effect and the ambipolar diffusion terms are not included in the induction equation given by Vögler et al. (2005) and used by Vögler and Schússler (2007) to study solar surface dynamo. In order to appreciate the time and the spatial scales over which the additional terms would contribute to the

magnetic induction evolution, the induction equation is written in a dimensionless form by using the normalizing quantities $B_0, L_0, t_0, V_0, n_{n0}$ such that $V_0 = L_0/t_0 = (B_0^2/4\pi\rho_n)^{1/2}$. The dimensionless induction equation reads:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \left[(\boldsymbol{V}_n - \epsilon_H \nabla \times \boldsymbol{B} + \epsilon_A (\nabla \times \boldsymbol{B}) \times \boldsymbol{B}) \times \boldsymbol{B} - \eta \nabla \times \boldsymbol{B} \right], \tag{17}$$

where

$$\epsilon_H = \frac{\lambda_i}{L_0} \left(\frac{\rho_n}{\rho_i} \right)^{1/2} \quad , \tag{18}$$

and $\lambda_i = \frac{c}{\omega_{ip}}$ is the ion inertial scale and $\omega_{ip} = (\frac{4\pi n_i e^2}{m_i})^{1/2}$ is the ion plasma frequency. Thus the Hall term brings in a finite spatial scale in an otherwise ideal MHD system of any arbitrary scale L. The coefficient ϵ_A of the ambipolar term is found to be:

$$\epsilon_A = \frac{\omega_{ic}}{\nu_{in}} \epsilon_H \ . \tag{19}$$

clearly highlighting the predominance of the ambipolar diffusion over the Hall effect for large $\boldsymbol{B}$ and small ν_{in}.

The mass conservation is described as:

$$\begin{aligned} \frac{\partial \rho_{\rm n}}{\partial t} + \nabla \cdot (\rho_{\rm n} \boldsymbol{V}_{\rm n}) = 0, \\ \nabla \cdot \boldsymbol{V}_{\rm i} = 0, \\ \nabla \cdot \boldsymbol{V}_{\rm e} = 0 \ . \end{aligned} \tag{20}$$

The energy equation for a partially ionized plasma is found to be:

$$\begin{aligned} & \frac{\partial \epsilon}{\partial t} + \nabla \cdot \left[\boldsymbol{V}_{\rm n} \left(\epsilon + p + \frac{\boldsymbol{B}^2}{8\pi} \right) \right. \\ & \left. + (V_{\rm H} + V_{\rm Am}) \frac{B^2}{8\pi} - \frac{(\boldsymbol{B}.\boldsymbol{V}_E)\,\boldsymbol{B}}{4\pi} \right] = \\ & -\nabla . \left[\frac{\eta (\nabla \times \boldsymbol{B}) \times \boldsymbol{B}}{4\pi} \right] + \\ & [\nabla . (\boldsymbol{V}_n . \tau) - s_{ij} \tau_{ij}] + \nabla . (\kappa \nabla T) + \\ & \rho_n (\boldsymbol{g}.\boldsymbol{V}_n) - \frac{4\pi\eta}{c^2} \boldsymbol{J}^2 + Q \ . \end{aligned} \tag{21}$$

where $s_{ij} = \frac{\partial V_{\rm ni}}{\partial x_{\rm j}}$. Here Q includes the rest of the nonadiabatic effects such as the radiative cooling. The energy equation explicitly displays the additional terms due to the Hall and the ambipolar effects. The effect of the ionization potential χ_i has not been taken explicitly into account in the internal energy. Instead one could continue with the methodology used

by Vögler et al. (2005) to consider pressure p and temperature T to be a function of the internal energy .

Equations (8), (13), (20) and (21) describe the solar surface dynamo.

3 THE ALPHA EFFECT IN COMPRESSIBLE THREE-COMPONENT TURBULENT MAGNETOFLUID

The α effect in an incompressible partially ionized plasma has been discussed recently (Krishan and Gangadhara 2008). Here an attempt is made to include the effect of turbulent density fluctuation of the neutral fluid along with the usual velocity and magnetic field fluctuations. The ambipolar diffusion term depends on the density of the neutral fluid through the ion-neutral collision frequency. The electron and the ion densities are kept constant and their fluctuations are ignored. Following the standard procedure (Krause & Rädler 1980) the velocity $\boldsymbol{V}_{\rm E}$, the magnetic field $\boldsymbol{B}$ and the neutral fluid density ρ_n are split into their average large scale parts and the fluctuating small scale parts as:

$$\boldsymbol{V}_{\rm E} = \overline{\boldsymbol{V}_{\rm E}} + \boldsymbol{V}'_{\rm E}, \tag{22}$$

$$\boldsymbol{B} = \overline{\boldsymbol{B}} + \boldsymbol{B}', \tag{23}$$

$$\rho_n = \overline{\rho_n} + \rho'_n \tag{24}$$

such that

$$\overline{\boldsymbol{V}'_{\rm E}} = 0, \qquad \overline{\boldsymbol{B}'} = 0, \qquad \overline{\rho'_n} = 0. \tag{25}$$

In the mean-field dynamo the magnetic induction equation is solved for large and small scale fields. Here, the mass conservation equation of the neutral fluid would also be considered. Substituting Eqs. (22-25) into the induction equation (13), we find, in the first order smoothing approximation,

$$\boldsymbol{V}'_{\rm E} = \boldsymbol{V}'_n - \frac{\boldsymbol{J}'}{en_{\rm e}} + \frac{\boldsymbol{J}'\times\overline{\boldsymbol{B}}}{c\nu_{\rm in}\rho_{\rm i}} + \frac{\overline{\boldsymbol{J}}\times\boldsymbol{B}'}{c\nu_{\rm in}\rho_{\rm i}} - \frac{\nu'_{in}}{c\rho_i\overline{\nu_{in}}^2}\overline{\boldsymbol{J}}\times\overline{\boldsymbol{B}} \,. \tag{26}$$

and the mean flow is found to be:

$$\overline{\boldsymbol{V}_{\rm E}} = \overline{\boldsymbol{V}_{\rm n}} - \frac{\overline{\boldsymbol{J}}}{en_{\rm e}} + \frac{\overline{\boldsymbol{J}}\times\overline{\boldsymbol{B}}}{c\overline{\nu_{\rm in}}\rho_{\rm i}} + \frac{\overline{\boldsymbol{J}'\times\boldsymbol{B}'}}{c\overline{\nu_{\rm in}}\rho_{\rm i}} - \frac{\overline{\overline{\boldsymbol{J}}\times\boldsymbol{B}'\nu'_{in}}}{c\rho_i\overline{\nu_{in}}^2} - \frac{\overline{\boldsymbol{J}'\times\overline{\boldsymbol{B}}\nu'_{in}}}{c\rho_i\overline{\nu_{in}}^2} \,. \tag{27}$$

The fluctuation in the ion-neutral collision frequency is defined as:

$$\nu'_{in} = A\rho'_n, \;. \tag{28}$$

$$A = \frac{\Sigma_{in}}{m_n}\left(\frac{8k_BT}{\pi m_{in}}\right)^{1/2} . \tag{29}$$

where the cross-section $\Sigma_{in} \sim 5 \times 10^{-15}$ cm^2 and $m_{in} = \frac{m_i m_n}{m_i + m_n}$ is the effective mass of the scattering particles.

The turbulent electromotive force Ξ is a function of the mean magnetic induction $\overline{\boldsymbol{B}}$ and mean quantities formed from the fluctuations, and is expressed as:

$$\Xi = \overline{\boldsymbol{V}'_{\rm E} \times \boldsymbol{B}'} = \alpha\overline{\boldsymbol{B}} - \beta\nabla\times\overline{\boldsymbol{B}} , \tag{30}$$

where

$$\begin{aligned} \alpha &= -\frac{\tau_{\rm cor}}{3}\overline{\boldsymbol{V}'_{\rm E}\cdot(\nabla\times\boldsymbol{V}'_{\rm E})} \\ &= \alpha_{\rm v} + \alpha_{\rm H} + \alpha_{\rm Am} . \end{aligned} \tag{31}$$

Here

$$\alpha_{\rm v} = -\frac{\tau_{\rm cor}}{3}\overline{\boldsymbol{V}'_{\rm n}\cdot\Omega'_{\rm n}} \tag{32}$$

is the measure of the average kinetic helicity of the neutral fluid in the turbulence possessing correlations over time τ_{cor}. Retaining only the first order contributions from the Hall and the ambipolar effects, the contribution to α from the Hall effect is found to be

$$\alpha_{\rm H} = \frac{2\tau_{\rm cor}}{3en_{\rm e}}\overline{\boldsymbol{J}'\cdot\boldsymbol{\Omega}'_{\rm n}} \tag{33}$$

The coupling of the charged components with the neutral fluid is clearly manifest through the possible correlation between the current density fluctuations and the vorticity fluctuations of the neutral fluid $\boldsymbol{\Omega}'_{\rm n} = \nabla\times\boldsymbol{V}'_{\rm n}$. The ambipolar term gives rise to

$$\alpha_{\rm Am} = \boldsymbol{\alpha}_{{\rm A}_0}\cdot\overline{\boldsymbol{B}} + \boldsymbol{\alpha}_{{\rm A}_1}\cdot\overline{\boldsymbol{J}} + \boldsymbol{\alpha}_{{\rm A}_2}\cdot\overline{\boldsymbol{J}}\times\overline{\boldsymbol{B}} , \tag{34}$$

with

$$\boldsymbol{\alpha}_{{\rm A}_0} = \frac{2\tau_{\rm cor}}{3c\rho_{\rm i}\overline{\nu_{\rm in}}}\overline{\boldsymbol{J}'\times\boldsymbol{\Omega}'_{\rm n}} , \tag{35}$$

$$\boldsymbol{\alpha}_{{\rm A}_1} = -\frac{2\tau_{\rm cor}}{3c\rho_{\rm i}\overline{\nu_{\rm in}}}\overline{\boldsymbol{B}'\times\boldsymbol{\Omega}'_{\rm n}} , \tag{36}$$

and

$$\boldsymbol{\alpha}_{{\rm A}_2} = \left(\frac{-A}{\overline{\nu_{in}}}\right)\frac{2\tau_{\rm cor}}{3c\rho_{\rm i}\overline{\nu_{\rm in}}}\overline{\boldsymbol{V}'_n\times\nabla\rho'_n} . \tag{37}$$

The contributions to α from the ambipolar diffusion with its essential nonlinear character manifest through its dependence on the average magnetic induction and its spatial variation. Note that the term $\boldsymbol{\alpha}_{{\rm A}_1}$ was ignored in Krishan and Gangadhara (2008) as therein only $\overline{\boldsymbol{B}}$

dependent terms were retained. Now that the terms proportional to $\overline{\boldsymbol{J}}\times\overline{\boldsymbol{B}}$ appear, it becomes necessary to retain terms proportional to $\overline{\boldsymbol{J}}$. The compressibilty effect appears through the correlation between the gradient of the density fluctuation and the velocity fluctuation and this term brings in a quadratic dependence on the large scale field $\overline{\boldsymbol{B}}$. One also observes that the Hall alpha (Eq. 33) requires a component of the fluctuating current density along the fluctuating vorticity of the neutral fluid whereas the ambipolar effect (Eq. 35) thrives on the component of the fluctuating current density perpendicular to the fluctuating vorticity. The turbulent dissipation parameter β is given by

$$\beta = \frac{\tau_{\rm cor}}{3}\overline{\boldsymbol{V}_{\rm E}'^2} = \beta_{\rm v} + \beta_{\rm H} + \beta_{\rm Am} \tag{38}$$

with

$$\beta_{\rm v} = \frac{\tau_{\rm cor}}{3}\overline{\boldsymbol{V}_{\rm n}'^2} \tag{39}$$

as the measure of the average turbulent kinetic energy of the neutral fluid in the turbulence possessing correlations over time $\tau_{\rm cor}$. The first order Hall contribution to β is

$$\beta_{\rm H} = -\frac{2\tau_{\rm cor}}{3en_{\rm e}}\overline{\boldsymbol{J}'\cdot\boldsymbol{V}_{\rm n}'} \tag{40}$$

The coupling of the charged components with the neutral fluid is clearly manifest through the possible correlation between the current density fluctuations and the velocity fluctuations of the neutral fluid. The ambipolar term furnishes

$$\beta_{\rm Am} = \boldsymbol{\beta}_{A0}\cdot\overline{\boldsymbol{B}} + \boldsymbol{\beta}_{A1}\cdot\overline{\boldsymbol{J}} + \boldsymbol{\beta}_{A2}\cdot\left(\overline{\boldsymbol{J}}\times\overline{\boldsymbol{B}}\right)\ , \tag{41}$$

where

$$\boldsymbol{\beta}_{\rm A0} = \frac{2\tau_{\rm cor}}{3c\rho_i\overline{\nu_{\rm in}}}\overline{\boldsymbol{V}_n'\times\boldsymbol{J}'}\ , \tag{42}$$

$$\boldsymbol{\beta}_{\rm A1} = \frac{2\tau_{\rm cor}}{3c\rho_i\overline{\nu_{\rm in}}}\overline{\boldsymbol{V}_n'\times\boldsymbol{B}'}\ , \tag{43}$$

$$\boldsymbol{\beta}_{\rm A2} = \left(\frac{-A}{\overline{\nu_{in}}}\right)\frac{2\tau_{\rm cor}}{3c\rho_i\overline{\nu_{\rm in}}}\overline{\boldsymbol{V}_n'\rho_n'}\ . \tag{44}$$

with its essential nonlinear character manifest through its dependence on the average magnetic induction $\overline{\boldsymbol{B}}$, the average current density $\overline{\boldsymbol{J}}$ and the average Lorentz force $\overline{\boldsymbol{J}}\times\overline{\boldsymbol{B}}$. One also observes that the Hall $\beta_{\rm H}$ requires a component of the current density fluctuations along the velocity fluctuations of the neutral fluid whereas the ambipolar effect thrives on the component of the current density fluctuations and the magnetic field fluctuations perpendicular to the velocity fluctuations in addition to the velocity- density correlation. We have used rigid or perfectly conducting boundary conditions (all surface contributions

vanish) while determining the averages. Here we consider what is known as the α^2 dynamo and take the mean flow $\overline{\boldsymbol{V}_{\mathrm{E}}} = 0$. This actually determines the relative mean flow amongst the three fluids. The dynamo equation reduces to

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times [\alpha \boldsymbol{B} - \beta \nabla \times \boldsymbol{B}] + \eta \nabla^2 \boldsymbol{B} \ . \tag{45}$$

The large scale magnetic field is written without the bar. The solutions of the dynamo equation for some representative cases were discussed in Krishan and Gangadhara (2008). Here the dynamo equation will be set up for the case where the neutral density fluctuations, through the ambipolar diffusion, are the major contributors to the α effect. Thus the dynamo equation becomes:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times [(\boldsymbol{\alpha_{\mathrm{A_2}}} \cdot \boldsymbol{J} \times \boldsymbol{B})\, \boldsymbol{B} - \beta_{\mathrm{A2}} \nabla \times \boldsymbol{B}] + \eta \nabla^2 \boldsymbol{B} \ . \tag{46}$$

The equation even in this reduced form is still quite complicated. In order to have a glimpse of the role of the ambipolar term through the density fluctuations, we ignore the dissipation term and estimate the characteristic timescale of evolution of the field to be:

$$t \approx \frac{L}{(\boldsymbol{\alpha_{\mathrm{A_2}}} \cdot \boldsymbol{J} \times \boldsymbol{B})} \ . \tag{47}$$

where

$$\alpha_{\mathrm{A_2}} = \frac{-2\tau_{\mathrm{cor}} \overline{\boldsymbol{V}_n' \times \nabla \rho_n'}}{4\pi \overline{\rho_n} \rho_i \overline{\nu_{in}}} \ . \tag{48}$$

The velocity-density correlation may be approximated by using the steady state mass conservation

$$\nabla \cdot \left[\left(\overline{\rho_n} + \rho_n' \right) \left(\overline{\boldsymbol{V}_n} + \boldsymbol{V}_n' \right) \right] = 0 \ . \tag{49}$$

Taking the average and assuming $\|\overline{\boldsymbol{V}_n' \cdot \nabla \rho_n'}\| \approx \|\overline{\rho_n' \nabla \cdot \boldsymbol{V}_n'}\|$, $\|\overline{\boldsymbol{V}_n} \cdot \nabla \overline{\rho_n}\| \approx \|\overline{\rho_n} \nabla \cdot \overline{\boldsymbol{V}_n}\|$ and $\|\overline{\boldsymbol{V}_n' \cdot \nabla \rho_n'} \approx \overline{\boldsymbol{V}_n' \times \nabla \rho_n'}\|$;

for the case of $\overline{\boldsymbol{V}_E} = 0$, retaining only the non-turbulent contribution, $\overline{\boldsymbol{V}_n} = \frac{\overline{\boldsymbol{J}}}{en_e} - \frac{\overline{\boldsymbol{J}} \times \overline{\boldsymbol{B}}}{c \overline{\nu_{in}} \rho_i}$. the characteristic timescale is found to be:

$$t \approx \tau_{cor} \frac{t_A^4 \nu_{in}^2}{\tau_{cor}^2} \frac{\rho_i^2}{\rho_n^2} \ . \tag{50}$$

where $t_A = \frac{L}{V_{nA}}$ is the Alfven crossing time of the characteristic spatial scale L and $V_{nA} = \frac{B}{\sqrt{4\pi \overline{\rho_n}}}$ is the Alfven speed of the neutral fluid. For the collisional plasma $t_A \nu_{in} > 1$ and t_A is also expected to be greater than τ_{cor} as well. Thus depending on the ion to neutral density ratio the magnetic field evolution may take place on time scale as short as τ_{cor}. This rather crude estimate of the time scale is at best a pointer towards a possible rapid evolution of the magnetic field due to ambipolar diffusion on the solar atmosphere.

4 CONCLUSION

In this first attempt at formulating the mean-field dynamo in a compressible partially ionized plasma, a complete set of multi-fluid- magnetic equations, in particular the Faraday law and the energy equations have been established with the hope that adequate computational resources would be deployed to understand this very important problem of the solar surface dynamo. Additional physics of the Hall and the ambipolar diffusion effects brings in a variety of spatial and time scales associated with the ion inertial scale, the gyrofrequencies and the collisional frequencies, the scales towards which the high resolution observations of the solar magnetic fields tend to lead us.

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REFERENCES

Alfvén H., Fälthammer C.-G., 1962, Cosmic Electrodynamics, Clarendon Press, Oxford

Brandenburg A., Zweibel E.G., 1994, ApJ, 427, L91

Chitre S.M., Krishan V., 2001, MNRAS, 323, L23

Krause F., Rädler K.-H., 1980, Mean Field Magnetohydrodynamics and Dynamo Theory, Pergman Press

Krishan, V., Gangadhara, R. T., 2008, MNRAS, 385, 489

Krishan V., Varghese A.B., 2008, Solar Physics, 247, 343

Leake J.E., Arber T.D., 2006, A&A, 450, 805

Vögler, A. and Schüssler, M., 2007, A&A

Vögler, A., Shelyag, S., Schüssler, M., Cattaneo, F., Emonet, T. and Linde, T. 2005, A&A, 429, 335